PMRF Symposium 2022
Cryptanalysis of KECCAK & Algorithms for Lattice problems

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Outline

KECCAK

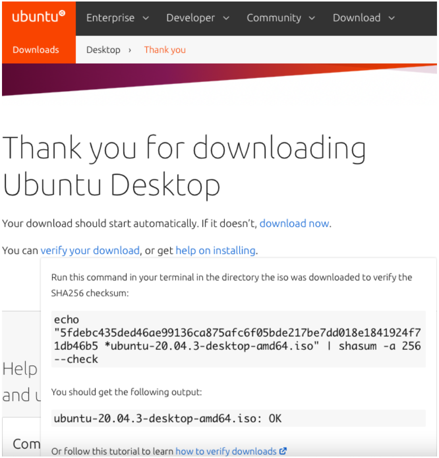

Figure: Snapshot of Ubuntu download page

Run this command in your terminal in the directory the iso was downloaded to verify the SHA256 checksum:

```
echo
"5fdebc435ded46ae99136ca875afc6f05bde217be7dd018e1841924f7
1db46b5 *ubuntu-20.04.3-desktop-amd64.iso" | shasum -a 256
--check
```

You should get the following output:

```
ubuntu-20.04.3-desktop-amd64.iso: OK
```

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- **SHA-3 (Secure Hash Algorithm 3)** is the latest member of the Secure Hash Algorithm family of standards, released by NIST which is based on **KECCAK**.

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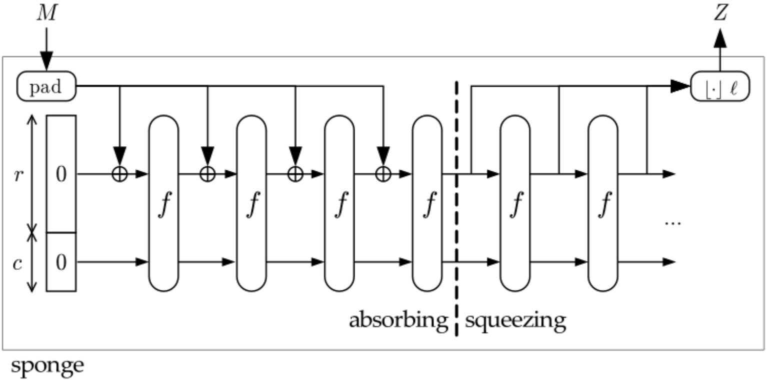
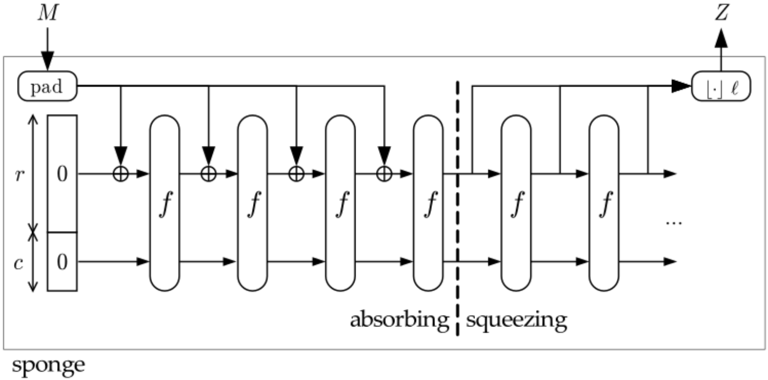
KECCAK: Sponge Construction



Source: http://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.202.pdf



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pad: padding function (10*1)

f: KECCAK-f permutation

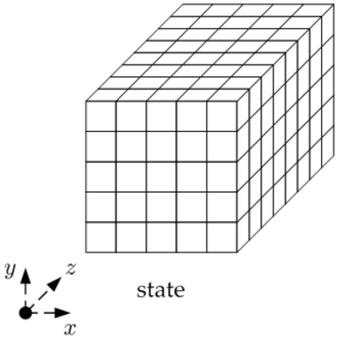


Figure: State

Source: https://keccak.team/figures.html

Description of θ

$S'[x,y,z] = S[x,y,z] \oplus P[(x+1) \ mod\ 5][(z-1) \ mod\ 64] \oplus P[(x-1) \ mod\ 5][z]$
where $P[x][z] = \bigoplus_{i=0}^{4} S[x,i,z]$

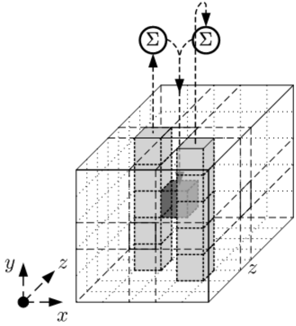


Figure: θ

Source: https://keccak.team/figures.html

Description of ρ

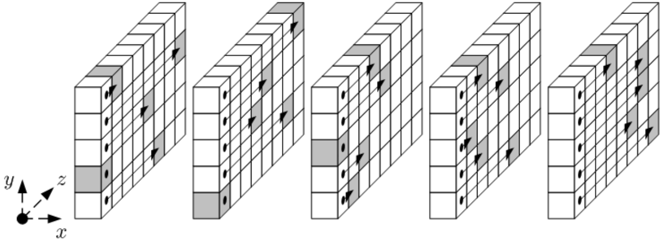
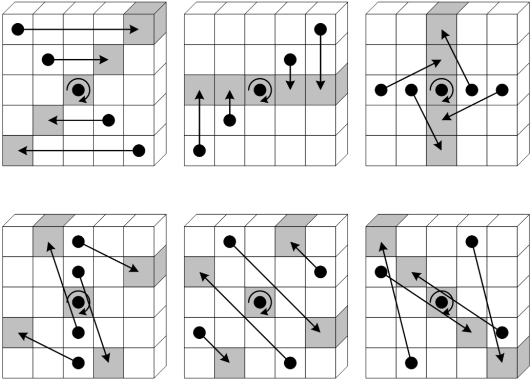


Figure: ρ

Source: https://keccak.team/figures.html

Description of π



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Description of χ and ι

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$$\begin{aligned} S'[x,y,z] = S[x,y,z] \oplus ((S[(x+1) \ mod\ 5, y, z] \oplus 1) \cdot \\ S[(x+2) \ mod\ 5, y, z]) \end{aligned}$$

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$$S'[0,0] = S[0,0] \oplus RC_i$$

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$$f = \underbrace{(\iota \circ \chi \circ \pi \circ \rho \circ \theta) \circ (\iota \circ \chi \circ \pi \circ \rho \circ \theta) \circ \cdots}_{r}$$

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Rounds	Instances	Our Results	Previous Results
2	384	$\mathbf{2^{113}}$	2^{129}[1]
	512	$\mathbf{2^{321}}$	2^{384}[1]
3	384	$\mathbf{2^{321}}$	2^{322}[1]
	512	$\mathbf{2^{475}}$	2^{482}[1]
4	384	$\mathbf{2^{371}}$	2^{378}[2]

Table: Summary of preimage attacks

$\mathbb{Z}^n$-isomorphism

- Most of the widely used cryptosystems like RSA, Diffie-Hellman and Elliptic Curve cryptosystem are based on integer factorization or discrete logarithm problem

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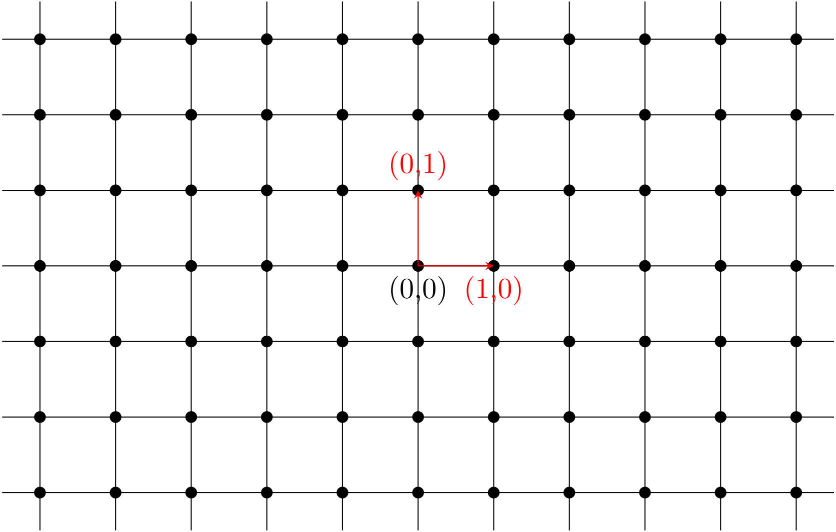
- These systems are vulnerable to quantum attacks that use the Shor's algorithm, which efficiently solves the above problems.
- Lattice based cryptosystems are one of the candidates for post-quantum cryptosystem. The security of such systems are based on the hardness of Shortest Vector Problem (SVP) and Closest Vector Problem (CVP).

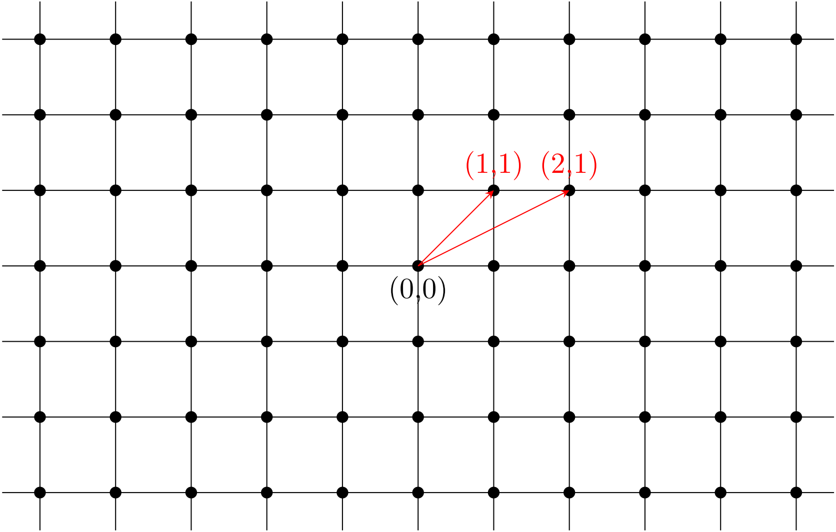
Let $B = [b_1, \ldots, b_n]$ is a set of linearly independent vectors. A **lattice** $\mathcal{L}(B)$ is the set of all integer linear combinations of the vectors in B, i.e.,

$$\mathcal{L}(B) = \{Bz | \forall z \in \mathbb{Z}^n\}$$

Lattice



$\mathbb{Z}^2$ lattice



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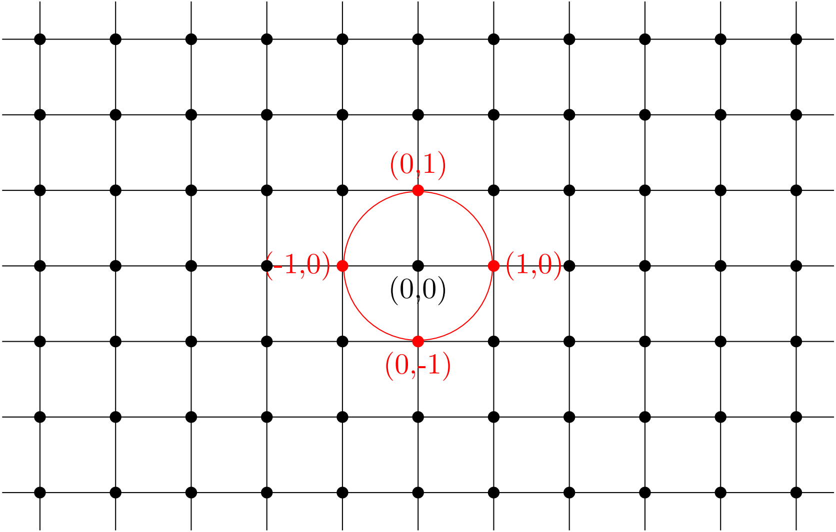
Shortest Vector Problem (SVP):- Given a lattice $\mathcal{L}(B)$, find a non-zero shortest vector v in $\mathcal{L}(B)$, i.e., $||v|| \leq ||w||, \forall w \in \mathcal{L}(B) \setminus \{0\}$.


(0,1)
(-1,0)
(1,0)
(0,0)
(0,-1)

Algorithm	**Time**	**Space**	**Deterministic/Randomized**
Enumeration [3]	$n^{O(n)}$	$poly(n)$	Deterministic
AKS [2]	$2^{O(n)}$	$2^{O(n)}$	Randomized
Voronoi based [1]	$\tilde{O}(2^{2n})$	$\tilde{O}(2^n)$	Deterministic
Gaussian Sampling [3]	$2^{n+o(n)}$	$2^{n+o(n)}$	Randomized

Table: Summary of SVP algorithms

$\mathbb{Z}^n$-isomorphism

Given a lattice $\mathcal{L}$, decide whether $\mathcal{L}$ is isomorphic to $\mathbb{Z}^n$.

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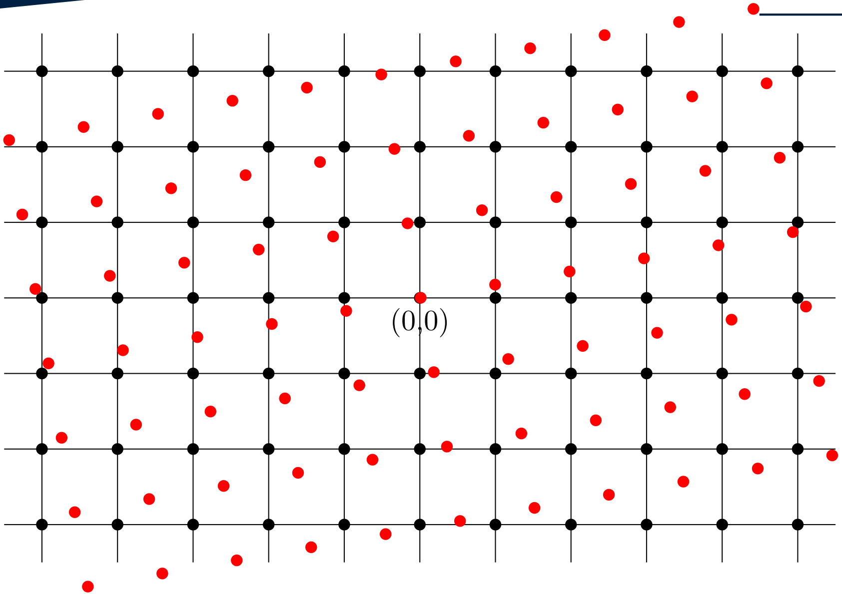
In other words, given a basis B, decide whether there exists an orthonormal matrix O and a unimodular matrix U such that

$$OB = U$$

Note: Any basis of $\mathbb{Z}^n$ is a unimodular matrix and vice-versa.

$\mathbb{Z}^2$ isomorphism



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- $\mathbb{Z}^n$-isomorphism can be solved using an SVP algorithm but it takes exponential time.

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2. $||b_i|| < ||v||, \forall i \in \{2, \ldots, n\}$.

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The proof of this theorem uses concepts from number theory.

Conjecture

Given a primitive vector $v_1, \ldots, v_k \in \mathbb{Z}^n$ such that

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2. $||b_i|| < ||v_1||, \forall i \in \{k+1, \dots, n\}$.

THANK YOU!

Rajasree M.S. (2019) Cryptanalysis of Round-Reduced KECCAK Using Non-linear Structures. In: Hao F., Ruj S., Sen Gupta S. (eds) Progress in Cryptology – INDOCRYPT 2019. INDOCRYPT 2019. Lecture Notes in Computer Science, vol 11898. Springer, Cham.
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