Incompressible Cryptography beyond Public Key Encryption

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Ongoing work with Rishab Goyal, Venkata Koppula, Aman Verma



Alice wants to send a message to Bob

s.t. no eavesdropper can learn the message





Alice wants to send a message to Bob

s.t. no eavesdropper can learn the message





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Decrypt using key k









































































































SKf







Alice wants to send m Parties learn only function of m







 SK_{f_2}







msk















msk

Alice wants to send m Parties learn only function of m







SK_{f3}

Learns only



Example attribute:



Example attribute:





Example access policy:

((Student AND CSE Dept) OR Dean)

Example attribute:



Encrypt messages with 'access policy'

Only users having attribute satisfying access policy should learn message



Example access policy:

((Student AND CSE Dept) OR Dean)

Alice wants to send m with policy f s.t. only parties whose attributes satisfy the policy can recover m

















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msk













SK_X.





 SK_{χ_2}



 X_{z}

 SK_{χ_3}



msk

ABE

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- FE for circuits, based on obfuscation [Garg, Gentry, Halevi, Raykova, Sahai, Waters 13]
- FE for circuits, based on bilinear maps + LWE [Jain, Lin, Sahai 20]

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What if decryption key is compromised?

MICROSOFT — DATA BREACH — AI — CYBERSECURITY — NEWS

Microsoft exposed 38TB of private AI data, including passwords and secret a tool for turning keys

Microsoft itself warns that it is "not possible to audit the generation of SAS tokens"

P

in

ED TARGETT

September 18, 2023 . 4:10 PM — 3 min read

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 \bowtie

 \bigcirc



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FORBES > INNOVATION > CYBERSECURITY

EDITORS' PICK

Davey Winder Senior Contributor ① Co-founder, Straight Talking Cyber

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Apr 28, 2020, 06:46am EDT



adversary to get the whole key

[Dziembowzki 06; Di Crescenzo, Lipton 06; Bellare, Kane, Rogaway 16]

- BIG KEY CRYPTOGRAPHY: make key so large that it is difficult for

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INCOMPRESSIBLE ENCRYPTION: this talk

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Adi Shamir

I want that the secret of the Coco-Cola exfiltrated.

company to be kept not in a tiny file of one kilobyte, which can be exfiltrated easily by an APT (Advanced Persistent Threat). I want that file to be a terabyte, which cannot be [easily]

(RSA 2013 conference)





















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- Can either try to learn recipe from the ciphertext, and send the recipe
- Can send a short summary of the ciphertext. Later, adversary learns key, and uses this summary to learn recipe.



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- FORWARD SECURE STORAGE [Dziembowski 06] Incompressible Secret Key Encryption
- INCOMPRESSIBLE CRYPTOGRAPHY [Guan, Wichs, Zhandry 22] Incompressible Public Key Encryption
- [Branco, Dottling, Dujmovic 22] Efficient incompressible PKE schemes from LWE/DDH

- RATE-1 INCOMPRESSIBLE ENCRYPTION FROM STANDARD ASSUMPTIONS

- DEFINE INCOMPRESSIBILITY FOR IBE/ABE/FE Multiple definitions possible - does adversary learn a distinguishing key, or the entire master secret key?

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- Optimal* Rate Constructions from Standard Assumptions

*Optimality lies in the eyes of the beholder.



Security Definitions, and connections to other crypto primitives

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- Incompressible SKE
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- Conclusion and Open Questions

Security Definitions
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Adversary



state

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Adversary wins if b = b'

Incompressible SKE

• $sk \in \{0,1\}^n$ Enc(sk,m): $ct = m \bigoplus sk$.

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- Consider $m_0 = 0^n$ and $m_1 = 1^n$. After receiving c, the adversary creates state = c[0].
- Only receiving sk, the second adversary returns $b' = state \oplus sk[0]$.

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- Output $c = (R, m \oplus sk')$.

Incompressible Security

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Incompressible Security Challenger Adversary 1




















































Our Incompressible PKE Scheme



• Primitives required - PKE, incompressible SKE and garbling scheme.



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 - Decrypt the incompressible SKE ciphertext.







Circuit Garbling







Correctness - For any *x*, $C(x) = \tilde{C}(\{lab_{i,x_i}\}).$



Circuit Garbling



$lab_{1,0}$ $lab_{2,0}$ $lab_{3,0}$ $lab_{1,1}$ $lab_{2,1}$ $lab_{3,1}$





Correctness - For any *x*, $C(x) = \tilde{C}(\{lab_{i,x_i}\})$. Security - \tilde{C} and $\{lab_{i,x_i}\}$ reveal C(x), but nothing else.



Circuit Garbling



 $lab_{1,0}$ $lab_{2,0}$ $lab_{3,0}$ $lab_{1,1}$ $lab_{2,1}$ $lab_{3,1}$





• *Setup(*): Generate 2*n* public/secret key, $(pk_{i,b}, sk_{i,b}) \leftarrow PKE.Setup()$ Generate $k \leftarrow incSKE$. Setup(). $pk = \{pk_{i,b}\} \text{ and } sk = (k, \{sk_{i,k_i}\})$



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- Enc(pk, m): $(\tilde{C}, lab_{i,b}) \leftarrow Garble(incSKE . Enc(\cdot, m))$ $c_{i,b} \leftarrow PKE . Enc(pk_{i,b}, lab_{i,b})$ Return (\tilde{C} , { $c_{i,b}$ })



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• $Dec(sk, (\tilde{C}, \{c_{i,b}\}))$: $lab_{i,k_i} \leftarrow PKE . Dec(sk_{i,k_i}, c_{i,k_i})$ $incSKE.ct = \tilde{C}(\{lab_{i.k.}\})$ $m \leftarrow incSKE.Dec(k, incSKE.ct)$ Return *m*



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Thank You!