Incompressible Functional Encryption

Mahesh Sreekumar Rajasree
CISPA Helmholtz

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Joint work with Rishab Goyal (UW-Madison), Venkata Koppula (IITD) and Aman Verma (IITD)

Functional Encryption (FE) [Sahai-Waters05…]

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mpk

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msk

mpk

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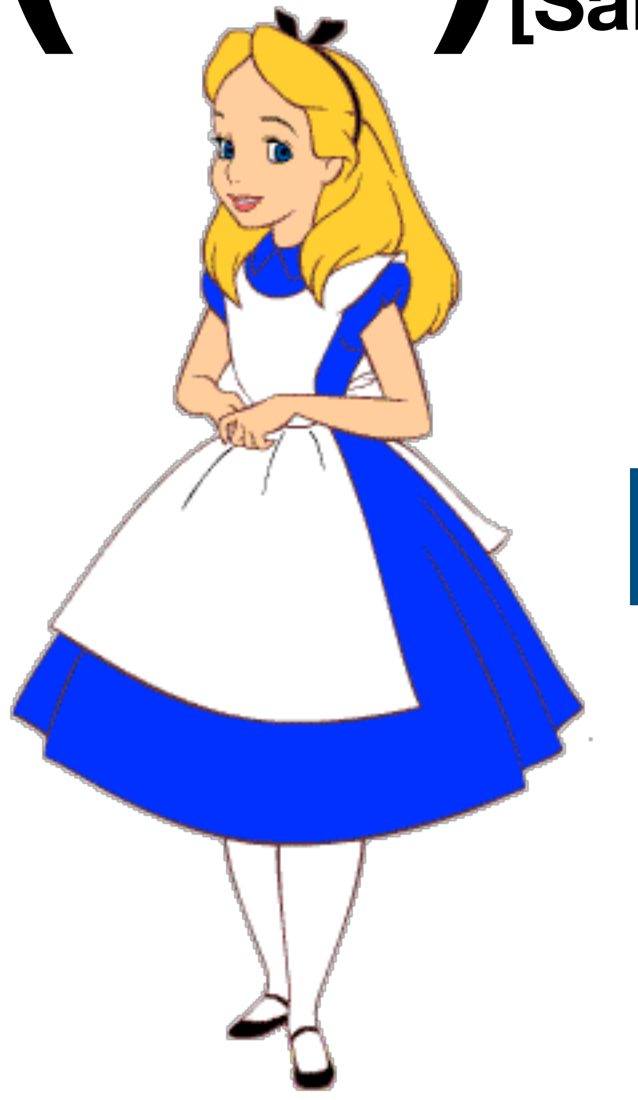
msk



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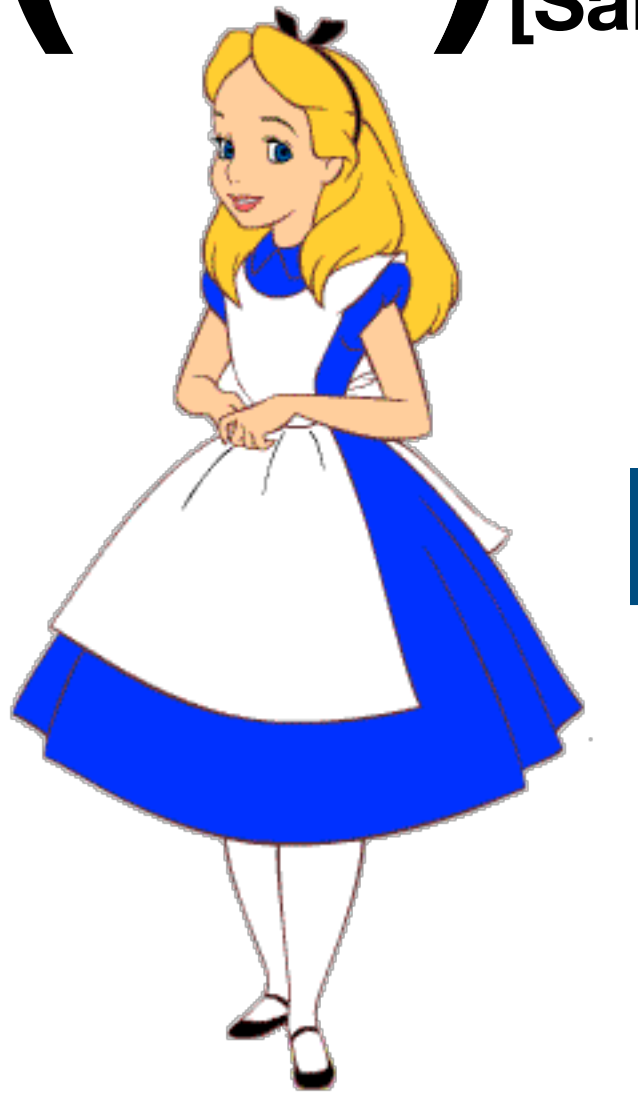
Functional Encryption (FE)[Sahai-Waters05…]

$\boxed{msk}$



$\boxed{mpk}, m$

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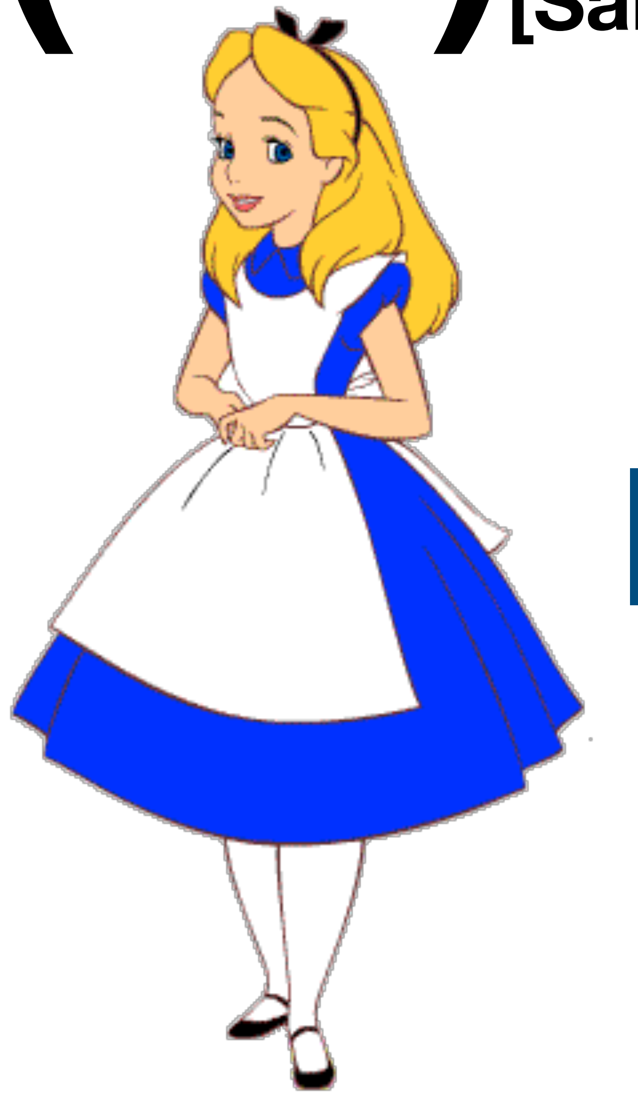

mpk, m

Functional Encryption (FE)[Sahai-Waters05…]

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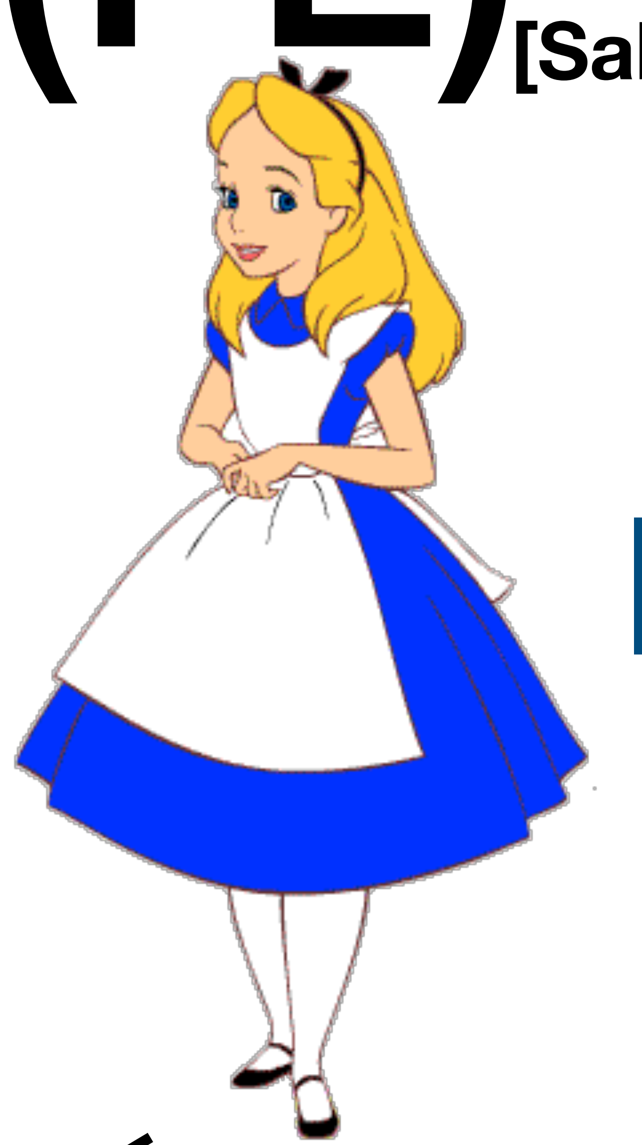

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sk_f

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- *Hides* everything but $f(m)$

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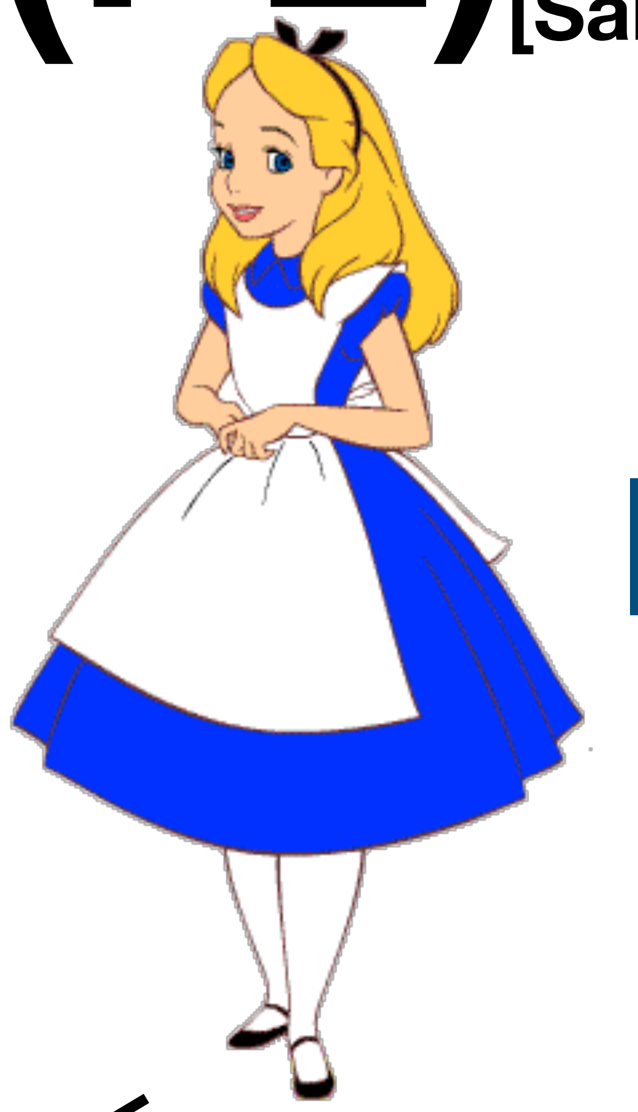
Functional Encryption (FE) [Sahai-Waters05…]

msk

- *Hides* everything but $f(m)$
- *Without* outsourcing to Bob
- Even when Bob is *offline*



mpk, m

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Learns only $f(m)$

FE Syntax

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$Setup(\lambda) \rightarrow$ master public key mpk , master secret key msk

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$Setup(\lambda) \to$ master public key $\boxed{mpk}$, master secret key $\boxed{msk}$

$Enc(\boxed{mpk}, m) \to$ Ciphertext $\boxed{ct}$

FE Syntax

$Setup(\lambda) \rightarrow$ master public key mpk, master secret key msk

$Enc(mpk, m) \rightarrow$ Ciphertext ct

$KeyGen(msk, f) \rightarrow$ Secret key sk_f

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$Enc(\boxed{mpk}, m) \rightarrow$ Ciphertext $\boxed{ct}$

$KeyGen(\boxed{msk}, f) \rightarrow$ Secret key $\boxed{sk_f}$

$Dec(\boxed{sk_f}, \boxed{ct}) \rightarrow f(m)$

FE Security

FE Security

Adversary

FE Security

$Enc(mpk, \mathbf{m_0})$

$sk_{f_1}, \ldots, sk_{f_q}$

Adversary

FE Security

$Enc(mpk, \mathbf{m_0})$
$sk_{f_1}, \ldots, sk_{f_q}$

Adversary

$Enc(mpk, \mathbf{m_1})$
$sk_{f_1}, \ldots, sk_{f_q}$

FE Security

$Enc(mpk, \mathbf{m_0})$
$sk_{f_1}, \ldots, sk_{f_q}$

Adversary

$Enc(mpk, \mathbf{m_1})$
$sk_{f_1}, \ldots, sk_{f_q}$

Indistinguishable whenever $f_i(m_0) = f_i(m_1)$ **for all** i

FE Security: Limitations

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- Master secret key must remain completely hidden from adversary.
  - Can generate any secret key!!!
- Wins if adversary obtains even a single **distinguishing key** (sk_f such that $f(m_0) \neq f(m_1)$).
  - Unrealistic to expect that **every secret key** can be securely stored.

Incompressible Cryptography

[Dziembowski'06,Guan-Wichs-Zhandry'22]

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- Security is lost if adversary has entire ciphertext and entire secret key due to correctness.

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  - Adversary gets a challenge ciphertext ct^* for m_0, m_1 and then it has to compress/reduce its storage which contains ct^*.

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  - Make ciphertext large so that long-term storage is expensive.
  - Adversary gets a challenge ciphertext ct^* for m_0, m_1 and then it has to compress/reduce its storage which contains ct^*.
  - After which it receives sk, but still should not be able to distinguish.

Prior works

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	Primitives

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Branco-Döttling-Dujmovic'23	Constructed CCA-Incompressible PKE (rate-1) from standard assumptions.
Guan-Wichs-Zhandry'23	Extended the notion to Multi-user Incompressible PKE setting.

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- Presented multiple incompressible FE schemes with (optimal) efficiency parameters.

This work

- Our goal — generalize incompressibility to Functional encryption.
  - Defined 3 levels of security notion.
  - Adversary can be provided either msk or multiple distinguishing keys or only a single distinguishing key.
- Presented multiple incompressible FE schemes with (optimal) efficiency parameters.
- Incompressible ABE from standard assumptions.

Incompressible FE Security

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Challenger

Adversary 1

Incompressible FE Security

Challenger

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$$(msk, mpk) \leftarrow Setup()$$

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$ $\xrightarrow{mpk}$

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$ $\xrightarrow{mpk}$

$\xleftarrow{f}$

$\xrightarrow{KeyGen(msk, f)}$

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$

mpk

f

$KeyGen(msk, f)$

m_0, m_1

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$ $\xrightarrow{\quad mpk \quad}$

$\xleftarrow{\quad f \quad}$ $\xrightarrow{\quad KeyGen(msk, f) \quad}$

$\xleftarrow{\quad m_0, m_1 \quad}$

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$ $\xrightarrow{mpk}$

$\xleftarrow{f}$ $\xrightarrow{KeyGen(msk, f)}$

$\xleftarrow{m_0, m_1}$

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$\xleftarrow{\quad f \quad}$ $\xrightarrow{\quad KeyGen(msk, f) \quad}$

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Incompressible FE Security

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$(msk, mpk) \leftarrow Setup()$ $\xrightarrow{\quad mpk \quad}$

$\xleftarrow{\quad f \quad}$ $\xrightarrow{\quad KeyGen(msk, f) \quad}$

$\xleftarrow{\quad m_0, m_1 \quad}$

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$ $\xrightarrow{\quad c \quad}$

$\xleftarrow{\quad f \quad}$ $\xrightarrow{\quad KeyGen(msk, f) \quad}$

$\xleftarrow{\quad state \quad}$

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$ $\xrightarrow{\quad mpk \quad}$

$\xleftarrow{\quad f \quad}$ $\xrightarrow{\quad KeyGen(msk, f) \quad}$

$\xleftarrow{\quad m_0, m_1 \quad}$

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$ $\xrightarrow{\quad c \quad}$

$\xleftarrow{\quad f \quad}$ $\xrightarrow{\quad KeyGen(msk, f) \quad}$

$\xleftarrow{\quad state \quad}$

$|state| \leq S$

Incompressible FE Security

Challenger

Adversary 1

Adversary 2

$(msk, mpk) \leftarrow Setup()$ $\quad mpk \rightarrow$

f $\leftarrow$ / $\rightarrow$ $KeyGen(msk, f)$

$\leftarrow m_0, m_1$

$b \leftarrow \{0,1\}$

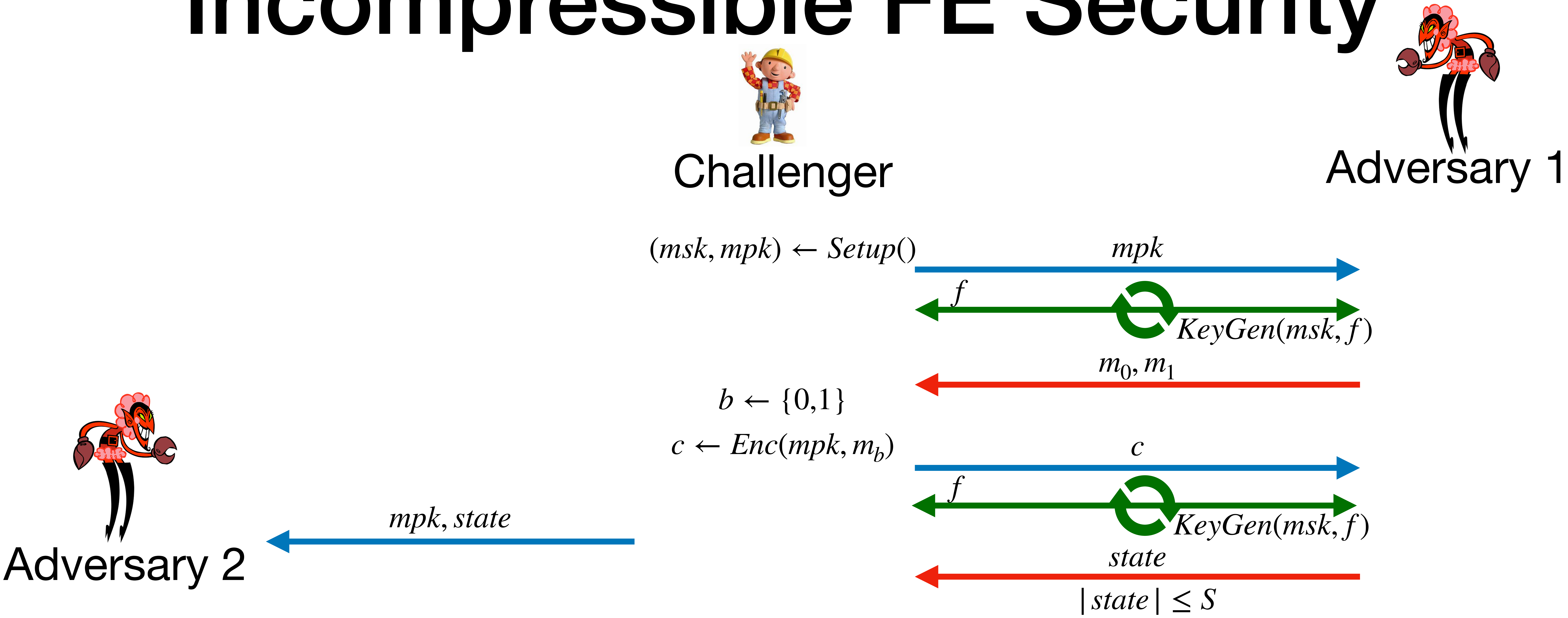
$c \leftarrow Enc(mpk, m_b)$ $\quad c \rightarrow$

f $\leftarrow$ / $\rightarrow$ $KeyGen(msk, f)$

$\leftarrow state$

$|state| \leq S$

Incompressible FE Security



Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$ $\quad$ mpk

f $\quad$ $KeyGen(msk, f)$

m_0, m_1

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$ $\quad$ c

f $\quad$ $KeyGen(msk, f)$

$state$

$|state| \leq S$

Adversary 2

$mpk, state$

$distinguishing\ f$

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$

mpk

f

$KeyGen(msk, f)$

m_0, m_1

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$

c

f

$KeyGen(msk, f)$

$state$

$|state| \leq S$

Adversary 2

$mpk, state$

distinguishing f

sk_f

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$ $\xrightarrow{mpk}$

$\xleftarrow{f}$ $\xrightarrow{}$ $KeyGen(msk, f)$

$\xleftarrow{m_0, m_1}$

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$ $\xrightarrow{c}$

$\xleftarrow{f}$ $\xrightarrow{}$ $KeyGen(msk, f)$

$\xleftarrow{state}$

$|\, state \,| \leq S$

Adversary 2

$\xleftarrow{mpk, state}$

$\xrightarrow{distinguishing\ f}$

$\xleftarrow{sk_f}$

$\xrightarrow{b' \in \{0,1\}}$

Incompressible FE Security

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$

mpk

f

$KeyGen(msk, f)$

m_0, m_1

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$

c

f

$KeyGen(msk, f)$

$state$

$|state| \leq S$

Adversary 2

$mpk, state$

$distinguishing\ f$

sk_f

$b' \in \{0,1\}$

Adversaries win if $b = b'$

Incompressible FE Security (Regular)

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$

mpk

f

$KeyGen(msk, f)$

m_0, m_1

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$

c

f

$KeyGen(msk, f)$

$state$

$|state| \leq S$

Adversary 2

$mpk, state$

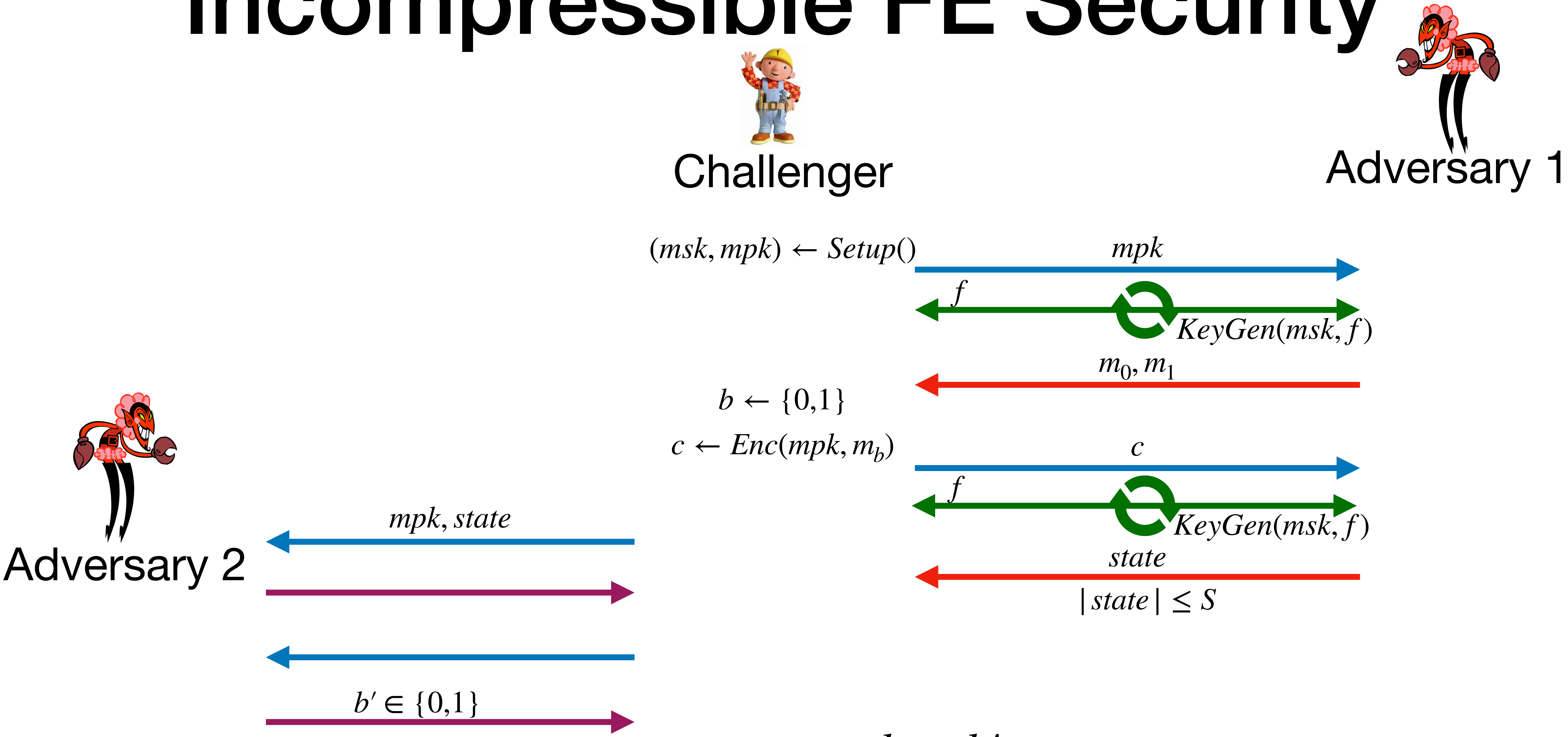
$distinguishing\ f$

sk_f

$b' \in \{0,1\}$

Adversaries win if $b = b'$

Incompressible FE Security



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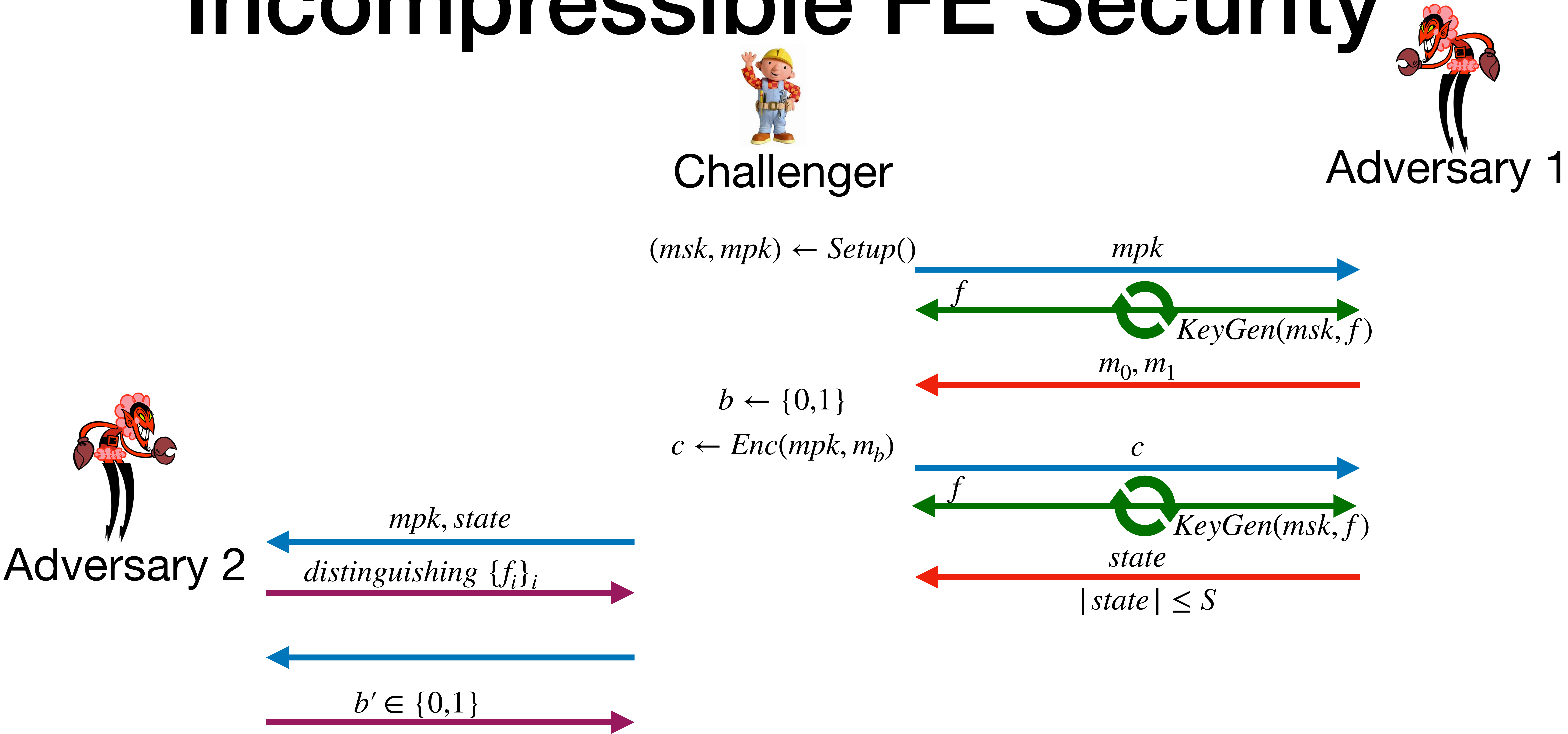
Incompressible FE Security



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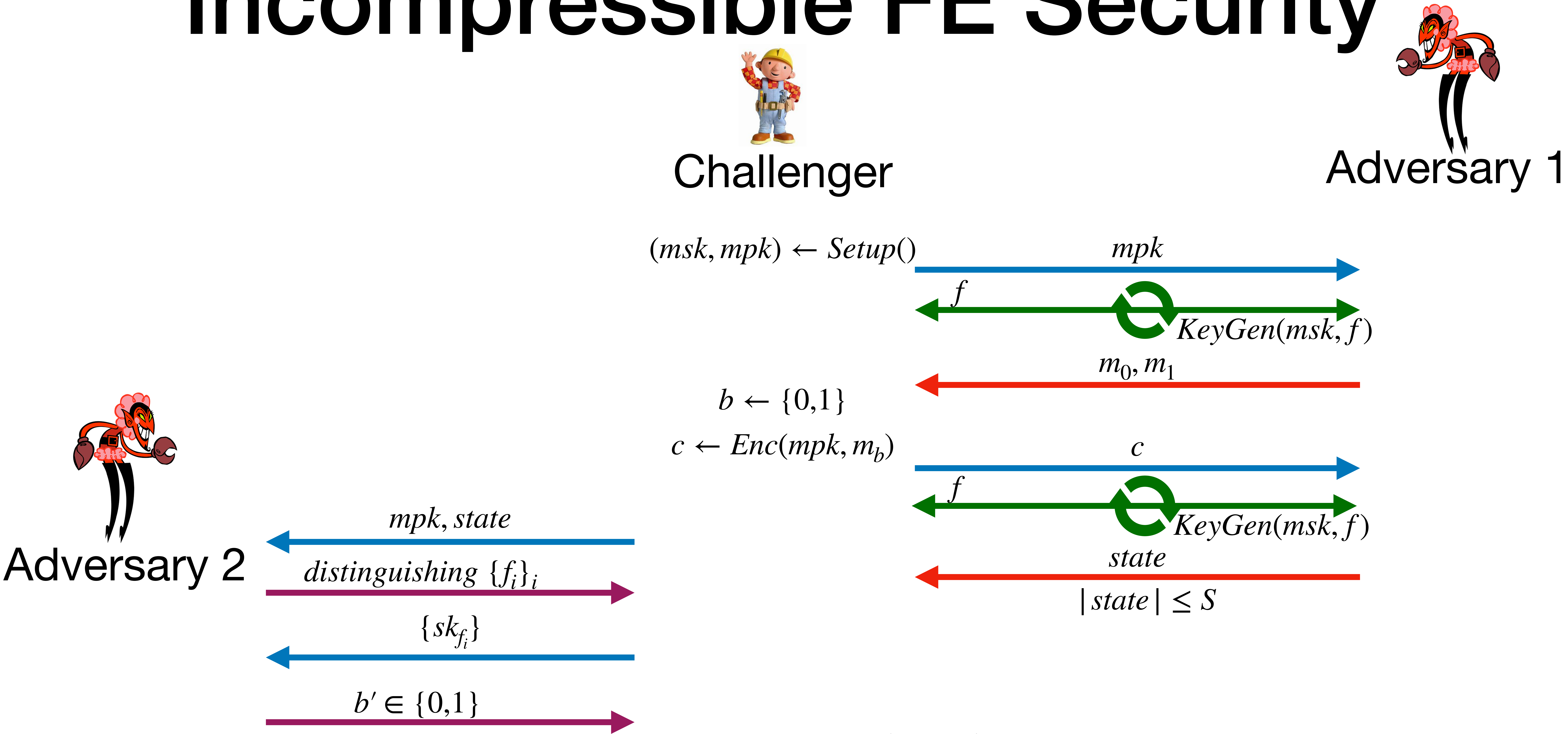


Adversaries win if $b = b'$

Incompressible FE Security
(Semi-Strong)

Challenger

Adversary 1

$(msk, mpk) \leftarrow Setup()$

mpk

f

$KeyGen(msk, f)$

m_0, m_1

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$

c

f

$KeyGen(msk, f)$

$state$

$|state| \leq S$

Adversary 2

$mpk, state$

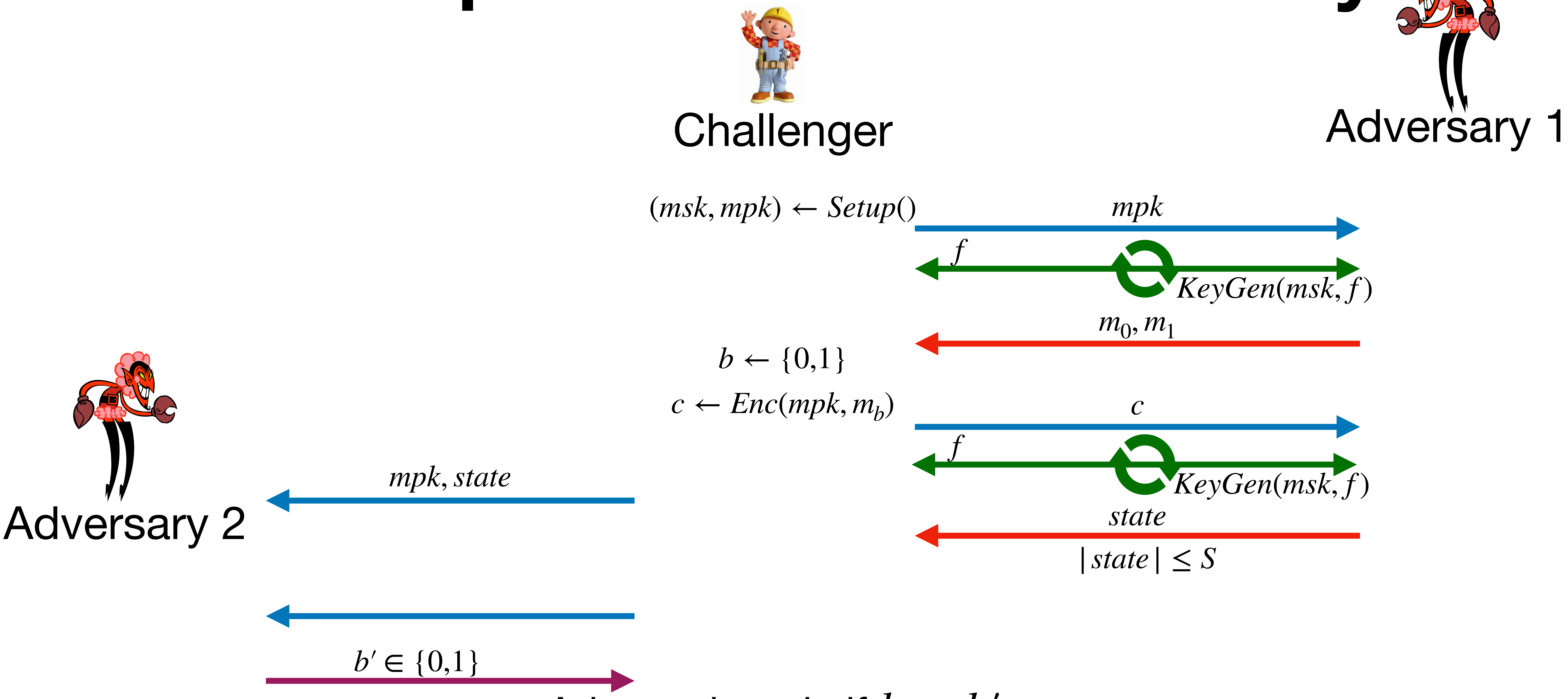
$distinguishing\ \{f_i\}_i$

$\{sk_{f_i}\}$

$b' \in \{0,1\}$

Adversaries win if $b = b'$

Incompressible FE Security



Adversaries win if $b = b'$

Incompressible FE Security

Challenger

Adversary 1

Adversary 2

$(msk, mpk) \leftarrow Setup()$

mpk

f

$KeyGen(msk, f)$

m_0, m_1

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c

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$state$

$|state| \leq S$

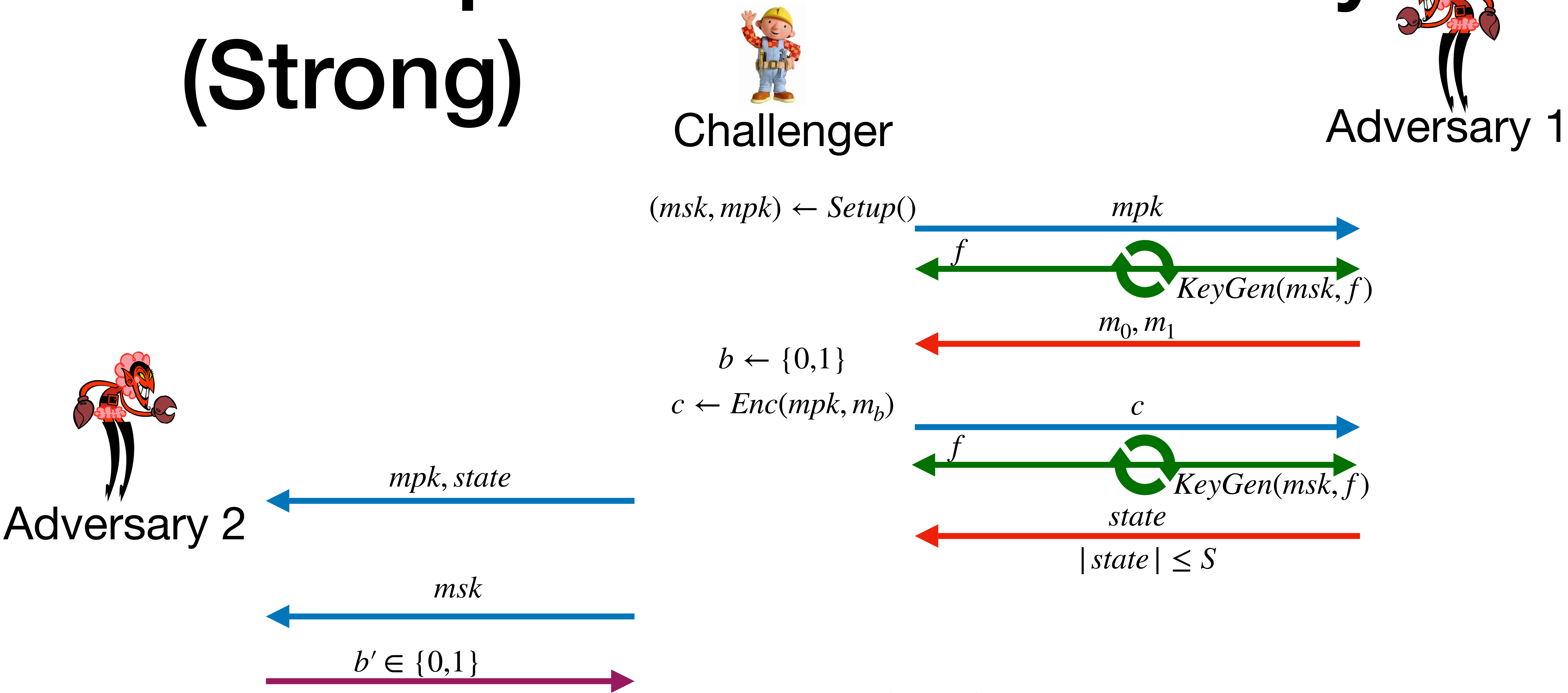
$mpk, state$

msk

$b' \in \{0,1\}$

Adversaries win if $b = b'$

Incompressible FE Security (Strong)



Adversaries win if $b = b'$

Our Results

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Primitive	Rate (|m| / |ct|)	Secret-key size	Adaptive	Assumptions

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Primitive	Rate (|m| / |ct|)	Secret-key size	Adaptive	Assumptions
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Regular Incomp FE	1	Short*	No	FE

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* = functions with one bit output

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OPTIMAL [BGKNPR'24]

* = functions with one bit output

Rate-1/2 Incomp FE with Large Keys

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1. **Regular FE scheme**

Rate-1/2 Incomp FE with Large Keys

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2. **Regular SKE scheme**

Rate-1/2 Incomp FE with Large Keys

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3. **Incompressible PKE scheme**

Rate-1/2 Incomp FE with Large Keys

Rate-1/2 Incomp FE with Large Keys

$Setup \rightarrow$

Rate-1/2 Incomp FE with Large Keys

$Setup \rightarrow MPK = ($

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\boxed{FE.MPK} \;,$$

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\boxed{FE.MPK}, \boxed{IncPKE.PK})$$

Rate-1/2 Incomp FE with Large Keys

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$$MSK = ($$

Rate-1/2 Incomp FE with Large Keys

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Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\ \boxed{FE.MPK}\ ,\ \boxed{IncPKE.PK}\ )$$

$$MSK = (\ \boxed{FE.MSK}\ ,\ \boxed{IncPKE.SK}\ )$$

$$Enc(m) \rightarrow$$

Rate-1/2 Incomp FE with Large Keys

$$Setup \to MPK = (\boxed{FE.MPK} , \boxed{IncPKE.PK})$$

$$MSK = (\boxed{FE.MSK} , \boxed{IncPKE.SK})$$

$$Enc(m) \to \qquad \boxed{IncPKE.Enc(\mathbf{0})}$$

Rate-1/2 Incomp FE with Large Keys

$Setup \rightarrow MPK = ($ $\boxed{FE.MPK}$, $\boxed{IncPKE.PK}$ $)$

$MSK = ($ $\boxed{FE.MSK}$, $\boxed{IncPKE.SK}$ $)$

$Enc(m) \rightarrow FE.Enc($ $\boxed{IncPKE.Enc(\mathbf{0})}$

Rate-1/2 Incomp FE with Large Keys

$Setup \rightarrow MPK = (\; FE.MPK \; , \; IncPKE.PK \;)$

$MSK = (\; FE.MSK \; , \; IncPKE.SK \;)$

$Enc(m) \rightarrow FE.Enc(\; m \; , \qquad IncPKE.Enc(\mathbf{0})$

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\ FE.MPK\ ,\ IncPKE.PK\ )$$

$$MSK = (\ FE.MSK\ ,\ IncPKE.SK\ )$$

$$Enc(m) \rightarrow FE.Enc(\ m\ ,\ 0\ ,\ IncPKE.Enc(\mathbf{0})$$

Rate-1/2 Incomp FE with Large Keys

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$$MSK = (\ \boxed{FE.MSK}\ ,\ \boxed{IncPKE.SK}\ )$$

$$Enc(m) \rightarrow FE.Enc(\ \boxed{m}\ ,\ \boxed{0}\ ,\ \boxed{IncPKE.Enc(\mathbf{0})}\ ,\ \boxed{\perp}\ )$$

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\boxed{FE.MPK}, \boxed{IncPKE.PK})$$

$$MSK = (\boxed{FE.MSK}, \boxed{IncPKE.SK})$$

$$Enc(m) \rightarrow FE.Enc(\boxed{m}, \boxed{0}, \boxed{IncPKE.Enc(\mathbf{0})}, \boxed{\perp})$$

$$KeyGen(f) \rightarrow$$

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\ FE.MPK\ ,\ IncPKE.PK\ )$$

$$MSK = (\ FE.MSK\ ,\ IncPKE.SK\ )$$

$$Enc(m) \rightarrow FE.Enc(\ m\ ,\ 0\ ,\ IncPKE.Enc(\mathbf{0})\ ,\ \perp\ )$$

$$KeyGen(f) \rightarrow FE.KeyGen(\ \hat{f}_{SKE.CT}\ )$$

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$$MSK = (\ FE.MSK\ ,\ IncPKE.SK\ )$$

$$Enc(m) \rightarrow FE.Enc(\ m\ ,\ 0\ ,\ IncPKE.Enc(\mathbf{0})\ ,\ \perp\ )$$

$$KeyGen(f) \rightarrow FE.KeyGen(\ \hat{f}_{SKE.CT}\ ) \longrightarrow SKE.Enc(\mathbf{0})$$

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\ \boxed{FE.MPK}\ ,\ \boxed{IncPKE.PK}\ )$$

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$$\hat{f}_{SKE.CT}($$

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$\hat{f}_{SKE.CT}(\boxed{m} ,$

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$$= \begin{cases} 1) \\ 2) \\ 3) \end{cases}$$

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$\hat{f}_{SKE.CT}(\boxed{m}, \boxed{b}, \boxed{CT}, \boxed{SKE.SK})$

$$= \begin{cases} 1) & \boxed{f(m)} \\ 2) & \\ 3) & \end{cases}$$

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$$\hat{f}_{SKE.CT}(\ m\ ,\ b\ ,\ CT\ ,\ SKE.SK\ )$$

$$= \begin{cases} 1) & f(m) \qquad \text{If } b = 0 \\ 2) & \\ 3) & \end{cases}$$

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\, FE.MPK \,,\, IncPKE.PK \,)$$

$$MSK = (\, FE.MSK \,,\, IncPKE.SK \,)$$

$$Enc(m) \rightarrow FE.Enc(\, m \,,\, 0 \,,\, IncPKE.Enc(\mathbf{0}) \,,\, \perp \,)$$

$$KeyGen(f) \rightarrow FE.KeyGen(\, \hat{f}_{SKE.CT} \,)$$

$$SKE.Enc(\mathbf{0})$$

$$\hat{f}_{SKE.CT}(\, m \,,\, b \,,\, CT \,,\, SKE.SK \,)$$

$$= \begin{cases} 1) & f(m) \\ 2) & \\ 3) & \end{cases}$$

If $b = 0$

Real Mode

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$\hat{f}_{SKE.CT}(\boxed{m}, \boxed{b}, \boxed{CT}, \boxed{SKE.SK})$

$$= \begin{cases} 1) & \boxed{f(m)} \\ 2) & \boxed{f(m)} \\ 3) & \end{cases}$$

If $b = 0$ $\longrightarrow$ Real Mode

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$$= \begin{cases} 1) & \boxed{f(m)} & \text{If } b = 0 \\ 2) & \boxed{f(m)} & \text{If } \hat{b} = 0 \\ 3) & & \end{cases}$$

Real Mode

$(\hat{b}, IncPKE.SK) = SKE.Dec(SKE.SK, SKE.CT)$

Rate-1/2 Incomp FE with Large Keys

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$$SKE.Enc(\mathbf{0})$$

$$\hat{f}_{SKE.CT}(\ m\ ,\ b\ ,\ CT\ ,\ SKE.SK\ )$$

$$= \begin{cases} 1)\ f(m) \\ 2)\ f(m) \\ 3)\ f(IncPKE.Dec(IncPKE.SK, CT)) \end{cases}$$

If $b = 0$

If $\hat{b} = 0$

Real Mode

$$(\hat{b}, IncPKE.SK) = SKE.Dec(SKE.SK, SKE.CT)$$

Rate-1/2 Incomp FE with Large Keys

$$Setup \rightarrow MPK = (\ FE.MPK\ ,\ IncPKE.PK\ )$$

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$$= \begin{cases} 1)\ f(m) & \text{If } b = 0 \\ 2)\ f(m) & \text{If } \hat{b} = 0 \\ 3)\ f(IncPKE.Dec(IncPKE.SK, CT)) & \end{cases}$$

Real Mode

$$(\hat{b}, IncPKE.SK) = SKE.Dec(SKE.SK, SKE.CT)$$

Distinguishing or not

Rate-1/2 Incomp FE with Large Keys

$Setup \rightarrow MPK = ($ $FE.MPK$, $IncPKE.PK$ $)$

$MSK = ($ $FE.MSK$, $IncPKE.SK$ $)$

$|ct| = |m| + 1 + |m| + poly(\lambda)$

$Enc(m) \rightarrow FE.Enc($ m , 0 , $IncPKE.Enc(\mathbf{0})$, $\perp$ $)$

$KeyGen(f) \rightarrow FE.KeyGen($ $\hat{f}_{SKE.CT}$ $)$ $\rightarrow$ $SKE.Enc(\mathbf{0})$

$\hat{f}_{SKE.CT}($ m , b , CT , $SKE.SK$ $)$

$$= \begin{cases} 1) & f(m) \\ 2) & f(m) \\ 3) & f(IncPKE.Dec(IncPKE.SK, CT)) \end{cases}$$

If $b = 0$ $\rightarrow$ Real Mode

If $\hat{b} = 0$ $\rightarrow$ Distinguishing or not

$(\hat{b}, IncPKE.SK) = SKE.Dec(SKE.SK, SKE.CT)$

Rate-1/2 Incomp FE with Large Keys

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$SKE.Enc(\mathbf{0})$

$\hat{f}_{SKE.CT}(\ m\ ,\ b\ ,\ CT\ ,\ SKE.SK\ )$

$$= \begin{cases} 1)\ f(m) \\ 2)\ f(m) \\ 3)\ f(IncPKE.Dec(IncPKE.SK, CT)) \end{cases}$$

If $b = 0$

If $\hat{b} = 0$

Real Mode

$(\hat{b}, IncPKE.SK) = SKE.Dec(SKE.SK, SKE.CT)$

Distinguishing or not

Extending to better parameters

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- Using rate-1/2 incompressible PKE and another layer of SKE encryption, secret keys can be made short.

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- Using rate-1/2 incompressible PKE and another layer of SKE encryption, secret keys can be made short.
- Replacing incompressible PKE component with extractors gives rate-1 but large keys.
- Small keys can be achieved if the functions are Boolean.

Incompressible ABE

Incompressible ABE

- Assuming the (sub-exp) hardness of LWE problem, there exists incompressible ABE for predicate classes with circuit of depth D with
  $|mpk| = poly(\lambda), \quad |sk| = poly(\lambda) \cdot D,$
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  - Uses two-level deferred encryption and this technique could find more applications in other contexts. Refer to the paper for more details.

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- From minimal assumption of ABE by extending ideas from Guan-Wichs-Zhandry'22.

Future Directions

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1. Rate-1 Semi-Strong Incompressible FE with adaptive security.

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2. Strong Incompressible FE with selective/adaptive security.

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1. Rate-1 Semi-Strong Incompressible FE with adaptive security.
2. Strong Incompressible FE with selective/adaptive security.
3. Strong Incompressible ABE/IBE from standard assumptions.
4. Using incompressible cryptography to build other primitives.

Thank You

https://eprint.iacr.org/2024/798.pdf